Write your name and student number. This exam comprises 3 problems. The total number of points is 34. The final grade is obtained by dividing the number of points by 3,4.

**Problem 1 (15 pts)**

A research reactor fuelled with U-235 is operated in a critical steady state at an average power of 40 MW. The shape of the reactor is a sphere with a diameter of 220 cm.

The properties of this uniform reactor are $\nu = 2.4$, $\Sigma_f = 0.0025\ cm^{-1}$ and $D = 1.1\ cm$.

a. Calculate the geometric buckling. (4 pts)
b. Calculate the diffusion length $L$. (4 pts)
c. If one fission releases $200\ MeV$ and $1\ MeV = 1.602\ 10^{-13}\ J$, what is the average value of the neutron flux? (3 pts)
d. Verify that the neutron flux is larger towards the middle of the reactor by calculating the flux at a radius of 10 cm and 100 cm. (4 pts)

**Solutions**

a. The geometric buckling for a sphere is given by

$$B^2 = \left(\frac{\pi}{R}\right)^2 = \left(\frac{\pi}{110}\right)^2 = 8.1567\ 10^{-4} cm^{-2}$$

b. The general expression for the buckling is

$$B^2 = \frac{1}{D}\left(\frac{\nu}{k}\Sigma_f - \Sigma_a\right)$$

$$B^2 D - \frac{\nu}{k}\Sigma_f = -\Sigma_a \Longrightarrow \Sigma_a = \frac{\nu}{k}\Sigma_f - B^2 D$$

Since the reactor is critical, we have $k = 1$. Replacing the value of $B^2$ obtained in a., we obtain

$$\Sigma_a = 2.4 \times 0.0025 - 8.1567\ 10^{-4} \times 1.1 = 5.1028\ 10^{-3} cm^{-1} = 0.0051\ cm^{-1}$$

The diffusion length is $L = \sqrt{D/\Sigma_a}$:

$$L = \sqrt{\frac{1.1}{0.0051}} = 14.7\ cm$$

c. The average power of a reactor is given by

$$P = E_f \times R_f \times V = E_f \times \Sigma_f \phi_{av} \times V$$

The volume of a sphere is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\ 110^3 = 5575279.8\ cm^3$$

The average flux is

$$\phi_{av} = \frac{P}{E_f \times V \times \Sigma_f} = \frac{40\ 10^6}{(200 \times 1.602\ 10^{-13})(5575279.8)(0.0025)}$$

$$\phi_{av} = 8.96\ 10^{13} cm^{-2} s^{-1}$$

d. The neutron flux at a radius $r$ in the spherical reactor is

$$\phi = A\frac{1}{r}\sin\left(\frac{\pi\ r}{R}\right)$$

With

$$A = \frac{P}{4R^2 E_f \Sigma_f} = \frac{40\ 10^6}{4(110^2)(200 \times 1.602\ 10^{-13})(0.0025)}$$

$$A = 1.03\ 10^{16}\ cm^{-1} s^{-1}$$

At a radius $r = 10\ cm$, we obtain

$$\phi = 1.03\ 10^{16}\frac{1}{10}\sin\left(\frac{\pi\ 10}{110}\right) = 2.91\ 10^{14}\ \ cm^{-2} s^{-1}$$

At a radius $r = 100\ cm$, we obtain

$$\phi = 1.03\ 10^{16}\frac{1}{100}\sin\left(\frac{\pi\ 100}{110}\right) = 2.91\ 10^{13}\ \ cm^{-2} s^{-1}$$

The values obtained at different radii show that the flux decreases with increasing radius.

**Problem 2 (11 pts)**

Consider a light-water moderated and cooled reactor. The fuel is uranium with an enrichment in U-235 of 3%.

a. What is the purpose of the moderator? (4 pts)
b. What is the purpose of the coolant? (3 pts)
c. Explain why is it necessary to enrich the fuel? (4 pts)

**Solutions**

a. The purpose of the moderator is to slow down the neutrons to thermal energies, as the fission cross section is largest for these thermal neutrons.
b. The purpose of the coolant is to transport the heat from the fission reactions to a steam generator.

c. Since water has a non-negligible neutron absorption cross section, the neutrons absorbed do not contribute to fission reactions. To compensate for this loss of neutrons, the amount of fissile material (U-235) needs to be increased.

**Problem 3 (8 pts)**

The neutron multiplication factor is $k = \varepsilon P_{FNL} p P_{TNL} f \eta$. Suppose that 1000 fast neutrons are created in the fuel.

a. The fast fission factor is $\varepsilon = 1.03$. How many new fast neutrons are created from fissions in the fuel? (3 pts)
b. The fast neutron non-leakage probability is $P_{FNL} = 0.96$. How many fast neutrons do not leak out of the reactor? (Round up the number) (2 pts)
c. 734 neutrons escape capture from the resonances. What is the resonance escape probability $p$? (3 pts)

**Solution**

a. The number of fast neutrons created in the fuel is $1000 \times \varepsilon = 1030$. Therefore, the number of new fast neutrons created by the fission of U-238 is $1030 - 1000 = 30$.
b. The number of fast neutrons not leaking out of the reactor is given by $n\varepsilon P_{FNL}$:
$1030 \times P_{FNL} = 989$.
c. The resonance escape probability is the ratio of the neutrons escaping capture to the number of fast neutrons still in the reactor. $p = 734/989 = 0.74$.

**Geometric buckling $B^2 (cm^{-2})$ :**

$$B^2 = \frac{1}{D}\left(\frac{\nu}{k}\Sigma_f - \Sigma_a\right)$$

**Diffusion length** $(cm)$:

$$L = \sqrt{\frac{D}{\Sigma_a}}$$

**Total average power of a reactor:**

$$P[W] = E_f \times R_f \times V = E_f \times \Sigma_f \phi_{average} \times V$$

$1\ MeV = 1.602\ 10^{-13}\ J$ and $1\ W = 1\ J/s$

**Buckling and neutron flux for different reactor geometries**

| Geometry | Dimensions | Buckling B² | Flux | A |
|---|---|---|---|---|
| Infinite slab | Thickness a | $\left(\frac{\pi}{a}\right)^2$ | $A\cos\left(\frac{\pi}{a}x\right)$ | $1.57 \times \frac{P}{a\,E_f\,\Sigma_f}$ |
| Rectangular parallepiped | a x b x c | $\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$ | $A\cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{b}y\right)\cos\left(\frac{\pi}{c}z\right)$ | $3.87 \times \frac{P}{V\,E_f\,\Sigma_f}$ |
| Infinite cylinder | Radius R | $\left(\frac{2.405}{R}\right)^2$ | $AJ_0\left(\frac{2.405\,r}{R}\right)$ | $0.738 \times \frac{P}{R^2 E_f \Sigma_f}$ |
| Finite cylinder | Radius R, height H | $\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$ | $AJ_0\left(\frac{2.405\,r}{R}\right)\cos\left(\frac{\pi\,z}{H}\right)$ | $3.63 \times \frac{P}{V\,E_f\,\Sigma_f}$ |
| Sphere | Radius R | $\left(\frac{\pi}{R}\right)^2$ | $A\,\frac{1}{r}\sin\left(\frac{\pi\,r}{R}\right)$ | $\frac{P}{4R^2\,E_f\,\Sigma_f}$ |

**Six-factor formula**: $k = \varepsilon p f \eta P_{FNL} P_{TNL}$

$\varepsilon$: fast fission factor:

$$\varepsilon = \frac{total\ number\ of\ fast\ neutrons}{number\ of\ fast\ neutrons\ \ produced\ by\ thermal\ fission}$$

$P_{FNL}$: fast (neutron) non-leakage probability:

$$P_{FNL} = \frac{number\ of\ fast\ neutrons\ that\ do\ not\ leak\ out}{number\ of\ fast\ neutrons\ produced\ by\ all\ fissions}$$

$p$: resonance escape probability (neutrons that pass through the U-238 resonances without being absorbed):

$$p = \frac{number\ of\ fast\ neutrons\ reaching\ thermal\ energy}{number\ of\ fast\ neutron\ that\ start\ to\ slow\ down}$$